Mathematical Analysis - List 18

1. Evaluate the given integral.

a) $\iint\limits_D \frac{dxdy}{(x+y+1)^3}$, $D = [0,2] \times [0,1]$;

b) $\iint\limits_D x \sin xy \, dxdy$, $D = [0,1] \times [\pi, 2\pi]$;

c) $\iint\limits_D e^{2x-y} \, dxdy$, $D = [0,1] \times [-1,0]$;

d) $\iint\limits_D xy \ln \frac{x}{y} \, dxdy$, $D = [1,e] \times [1,2]$.

2. Let f be a continuous function on D that is bounded by the given curves. Change $\iint\limits_D f(x,y) \, dxdy$ to an iterated integral.

a) $x^2 + y = 2$, $y^3 = x^2$;

b) $x^2 + y^2 = 4$, $y = 2x - x^2$, $x = 0$ $(x, y \geqslant 0)$;

c) $x^2 - 4x + y^2 + 6y - 51 = 0$;

d) $x^2 - y^2 = 1$, $x^2 + y^2 = 3$, $(x < 0)$.

3. Reverse the order of integration:

a) $\int\limits_{-1}^{1} dx \int\limits_{0}^{|x|} f(x,y) \, dy$;

b) $\int\limits_{-1}^{1} dx \int\limits_{-\sqrt{1-x^2}}^{0} f(x,y) \, dy$;

c) $\int\limits_{0}^{4} dx \int\limits_{-\sqrt{4x-x^2}}^{2\sqrt{x}} f(x,y) \, dy$;

d) $\int\limits_{1}^{e} dx \int\limits_{\ln x}^{1} f(x,y) \, dy$.

4. Evaluate the integral by reversing the order of integration.

a) $\int\limits_0^1 \int\limits_y^1 e^{x^2} \, dxdy$;

b) $\int\limits_0^3 \int\limits_{y^2}^9 y \sin(x^2) \, dxdy$;

c) $\int\limits_0^1 \int\limits_{\sqrt{y}}^1 \sqrt{x^3 + 2} \, dxdy$

5. Set up, but do not evaluate, an iterated integral for the volume of the solid.

a) Under the graph of $f(x,y) = 25 - x^2 - y^2$ and above the xy-plane.
b) Below the graph of $f(x,y) = 25 - x^2 - y^2$ and above the plane $z = 16$.
c) The three-sided pyramid whose base is on the xy-plane and whose three sides are the vertical planes $y = 0$ and $y - x = 4$, and the slanted plane $2x + y + z = 4$.

6. Convert the integrals to polar coordinates and evaluate.

a) $\iint\limits_D xy \, dxdy$, $D : x \geqslant 0$, $1 \leqslant x^2 + y^2 \leqslant 2$;

b) $\iint\limits_D y^2 e^{x^2+y^2} \, dxdy$, $D : x \geqslant 0$, $y \geqslant 0$, $x^2 + y^2 \leqslant 1$;

c) $\iint\limits_D (x^2 + y^2) \, dxdy$, $D : y \geqslant 0$, $y \leqslant x^2 + y^2 \leqslant x$;

d) $\int\limits_0^{\sqrt{2}} dy \int\limits_y^{\sqrt{4-y^2}} xy \, dx$;

e) $\int\limits_0^{\sqrt{6}} dx \int\limits_{-x}^{x} dy$.